Engineering Notes

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A Simple Estimate of the Effect of Ejector Length on Thrust Augmentation

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Nomenclature

F = $wJ_2(L)/\delta(L)J_1^2(L)$
$J_1(x)$ = $(\pi/4)^{1/2}\text{erf}[w/\delta(x)]$
$J_2(x)$ = $(\pi/8)^{1/2}\text{erf}[2^{1/2}w/\delta(x)]$
k = jet spreading constant
L = length of mixing duct
p = local static pressure
P_T = total pressure
t = half thickness of primary nozzle
u = velocity
w = half width of mixing duct
x,y = coordinates respectively aligned with and normal to the direction of flow and originating at the center of the exit plane of the primary nozzle
$\delta(x)$ = local length scale of jet
η = $y/\delta(x)$
ρ = mass density of fluid
Φ = thrust augmentation ratio, Eq. (7)

Subscripts

c = evaluated on ejector plane of symmetry
e = fluid with Bernoulli constant $P_{T\infty}$
isen = expanded from $(\ )_0$ to $(\ )_\infty$ conditions
0 = primary fluid
1 = inlet conditions
∞ = ambient conditions

ENGINEERS recently attracted to modern ejector technology frequently question the precise role played by the turbulent mixing process. Other engineers, those who have followed the algebra of von Karman's[1] momentum balance and who have begun designing ejectors, often lose sight of penalties associated with designs that prevent nearly complete mixing between the primary and entrained streams. A third class of engineers, those already armed with some ejector experience, would like to know the effects of accelerated mixing, or hypermixing, on ejector performance.

This Note is intended to provide qualitative answers for those described. The analysis is much more descriptive than it is accurate and relies more on physical modeling than on the precise solution to the equations of motion.

Consider the ejector sketched in Fig. 1 and for the time being neglect losses. Mixing between the incompressible primary and secondary streams takes place in a planar duct of width $2w$ and unit span. The confined mixing reduces the duct pressure and entrains ambient air with a Bernoulli constant of $P_{T\infty}$. The secondary stream is assumed to be uniformly distributed with a speed u_1 at the plane of injection, station 1. Mixing transforms the top hat profile of station 1 to the more rounded profile of station A. At some distance downstream from the plane of injection, the velocity profile can be approximated by

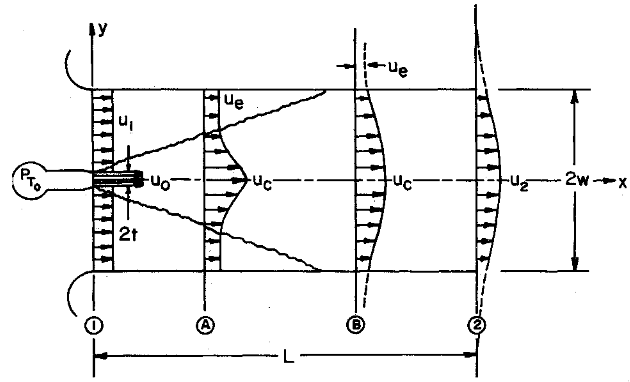

Fig. 1 Ejector.

$$u = u_e + (u_c - u_e)e^{-(y/\delta)^2} \quad (1)$$

evaluated within the duct, $-w \leq y \leq w$. Similar profiles have been used to describe jets in coflowing streams[2] and should be equally descriptive of confined jet mixing, provided the ratio w/t is not too close to unity. δ is a length scale that characterizes the width of the primary jet and increases with downstream distance. Fluid with velocity u_e, still untouched by the primary jet, can be related to the local pressure through the Bernoulli equation

$$\rho u_e^2/2 + p = P_{T\infty} = \rho u_1^2/2 + p_1 \quad (2)$$

At station B, further downstream, the primary jet has spread to the walls of the duct, but the profile can still be represented by Eq. (1) within the region $|y| \leq w$. From this point downstream, u_e is really a ficticious quantity, but could still be used to compute p. At the end of the duct the pressure is atmospheric, so $u_e = 0$ from Eq. (2).

How the length L of the duct affects the amount of fluid entrained into the ejector, and thus its thrust performance, can be demonstrated by using Eqs. (1) and (2) in

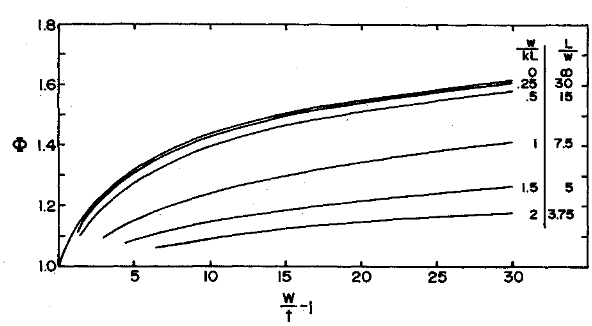

Fig. 2 Calculation of thrust augmentation Φ as a function of inlet area ratio $(w/t - 1)$ with ejector length to width ratio L/W as parameter. k is taken as 0.133.

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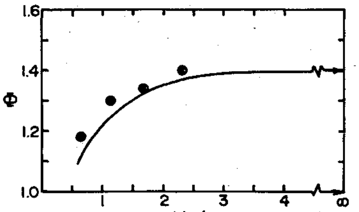

Fig. 3 The effect of kL/W on the performance of an inlet area ratio 8 ejector. Data points taken from Ref. 6.

a balance of the mass

$$\rho t u_o + \rho(w - t)u_1 = \delta(x)\rho \int_0^{w/\delta(x)} [u_e + (u_c - u_e)e^{-\eta^2}]d\eta \quad (3)$$

and impulse

$$p_1 w + \rho u_0^2 t + \rho u_1^2(w - t) = p(x)w + \delta(x)\int_0^{w/\delta(x)} [u_e + (u_c - u_e)e^{-\eta^2}]^2 d\eta \quad (4)$$

between station 1, the plane of injection and an arbitrary downstream station, x. The pressure has been assumed constant across the planes in question. After some algebraic manipulation the result is

$$(u_e/u_1)^2[1/2 + \delta(x)(J_2 - 2J_1)/w] - 2(u_e/u_1)(J_2/J_1 - 1)((u_0\delta(x)/u_1 w) + 1 - t/w) + (w/\delta(x))(J_2/J_1^2)((u_0\delta(t)/u_1 w) + 1 - t/w)^2 + (t/w)(1 - (u_0/u_1)^2) - 1/2 = 0 \quad (5)$$

An attempt to start at the injection plane and numerically solve Eq. (5) while marching downstream would result in failure for several reasons, not the least of which is the inadmissibility of Eq. (1) at small values of x. It would also defeat the purpose of this Note. The required relation between ejector length and thrust performance can be obtained by evaluating Eq. (5) at the exit plane of the ejector, station 2, where $u_e = 0$. The simplified equation can be written

$$(t/w)^2(1 - u_1/u_0)^2 - 2(t/w)[(1 - (u_1/u_0)^2)/(2F) - u_1/u_0 + (u_1/u_0)^2] + (u_1/u_0)^2[1 - 1/(2F)] = 0 \quad (6)$$

Eq. (6) is a simple quadratic in t/w that can be solved on a desk top calculator when the local value of $\delta(x)$ is specified. Some experimental data[3] have been successfully correlated by $\delta(x)/x = k(u_0 - u_1)/(u_0 + u_1)$. k is an experimental constant relating an appropriate dimension of the primary nozzle to its area, and is approximately equal to 0.133 for planar jets and nearly double that for the hypermixing jets used in Refs. 3-5. The solution of Eq. (6) associates the appropriate value of t/w with prescribed values of u_1/u_0 and wk/L. The thrust augmentation ratio Φ, can then be computed from:

$$\Phi = \int_{-w}^{w} \rho u^2 dy/2\rho t u_0 u_{1\text{sen}} = F(w/t)[t/w + (u_1/u_0)(1 - t/w)]^2/[1 - (u_1/u_0)]^{1/2} \quad (7)$$

Typical calculations are plotted in Fig. 2 and clearly point out the reduction in augmentation with increases in w/kL, or equivalently, decreases in L. It is obvious that hypermixing nozzles increase the augmentation of an ejector of fixed dimensions only by effectively decreasing the w/L ratio. Designers will be interested in noting that, in the absence of losses, the performance of an ejector whose length is 30 times its width differs little from an infinitely long ejector. Nearly the same can be said for an ejector of half that length, but a further reduction by two in length would be intolerable. In general, good performance can be expected from ejectors designed with $kL/w \geq 2.5$. Figure 3 shows that further increasing the length affords little improvement in augmentation and would, in fact, degrade performance when friction losses are included. Also shown are some data from Ref. 6, modified for consistency with Φ defined by Eq. (7). Lobes of thickness 0.108 in. on the experimental nozzle produced an effective nozzle thickness $2t$ = 0.453 in., and suggested taking k = (0.133) (0.108)/(0.453) = 0.0318. Diffusion provided by the slightly (5°) diverging walls of the experiment produced augmentation ratios greater than predicted by the present very simple calculation. The trend is nevertheless indicated successfully.

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Remarks on Vortex-Lattice Methods

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Introduction

ONE of the methods that has been applied to the solution of steady lifting-surface problems is the so-called vortex-lattice technique. This approach developed from the work of Falkner,[1] Rubbert,[2] and others, and has proven to be remarkably successful in treating a variety of configurations. An associated extension, the doublet-lattice technique, has been used to treat the corresponding unsteady flows.[3]

This Note presents the results of some numerical experiments on simple planar configurations which serve to establish more precisely some ground rules for optimum lattice arrangements. In particular, the location of both the horseshoe vortex elements and control points at which the

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